Making Work Work

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Some difficulties that students have in dealing with work and energy can be addressed through energy flow diagrams and energy bar graphs which allow students to represent features of systems and processes that $W = F\Delta x \cos\theta$ cannot. These representations are applied to various physical situations.

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Introduction

In many introductory physics texts the concept of work is separated from the concept of *energy* and from the concept of *system* in such a way that certain difficulties arise. Work is a carefully defined and subtle concept that is dependent on and derived from the energy concept and the system concept. The purposes of this paper are to identify certain difficulties that arise from common treatments of work and then to present some representational tools that can help to prevent these difficulties.

Difficulties with common approaches to energy

Often work is defined as

$$W = F\Delta x\cos\theta \tag{1}$$

where $F\cos\theta$ is the component of the constant force F acting on some object in the direction of the one dimensional displacement of the point of application of the force, Δx. Three problems for students can arise when this definition of work is not closely linked to the concept of energy. They are: 1) the belief that *forces* do work, 2) the belief that every force acting to some extent along the direction of an object's motion must do work on the object, and 3) the belief that the work done by an agent exerting frictional forces can be calculated by equation (1). Sadly, the understanding that work is an amount of *energy* that flows into or out of a system can be lost on students who know work only as $W = F\Delta x\cos\theta$.

The claim that forces themselves do work arises from the separation of the concept of work from the first law of thermodynamics,

$$\Delta E = Q - W \tag{2}$$

in which ΔE is the change in energy of a *system*, Q is the energy gained by the system through heating, and W is the energy lost from the system through working. A good understanding of $\Delta E = Q - W$ informs us that work has meaning only with respect to explicitly defined systems. This is not clear from $W = F\Delta x\cos\theta$ alone. Only systems that can contain and exchange energy can perform work. Forces are not systems and cannot contain or exchange energy, and, therefore, should not be said to perform work.

Another problem with thinking of work only as $W = F\Delta x\cos\theta$ is that it does not *explicitly* represent an exchange of energy from one *system* to another. Therefore, questions like "How much work is done when a 50 kg person climbs a 100 m hill?" are not specific enough to answer. Are we talking about work done by the gravitational field or by the person? The work done by the person is always greater than the energy gained by the gravitational field, so it makes a difference. We need to enable our students to build a better understanding of physical situations than that fostered by such questions. Because the quality of our understanding depends on the quality of our representational tools, it will be of benefit to students to have ways to represent systems and the flow of energy between and within systems. Two useful ways to represent energy exchanges have already been developed and will be described below. They are energy bar graphs and energy flow diagrams. After the students are acquainted with energy exchanges through the use of such representations, the concepts of working and heating can then be understood as particular kinds of already familiar exchanges.

Another difficulty common among students is the belief that work is done through *any* force on an object acting at least somewhat in the direction of its motion. For an example of a situation in which this belief is misleading consider a self-propelled object such as a jumping disk. The jumping disk is "cocked" into position and placed upon a table. No energy enters the jumping disk from the outside world as it "pops" up into the air. It does move upward, because the table exerts a force on it that causes it to accelerate. But there is no work done on the disk. There is simply a redistribution of energy within the disk itself. A careless application of $W = F\Delta x\cos\theta$ or the work-energy theorem

($\Delta E_{kinetic} = W_{net}$) will lead students to the conclusion that there *is* work done on the disk. There is *no* work done on the disk.

Yet another problem with the work concept actually is sometimes even encouraged by textbooks. Work done by agents exerting frictional forces cannot be calculated by $W = F\Delta x\cos\theta$. Despite this fact, many textbooks actually assert that it *can* be calculated this way. The problem here is our limited knowledge. We do not know the values of the actual magnitudes of the forces being exerted on the microscopic level at the points of contact, nor do we know the distances over which those forces are exerted. Textbooks often use the *net* frictional force and the center of mass displacement Δx_{cm} as though they are the appropriate variables to insert into $W = F\Delta x\cos\theta$. They are not. Therefore students end up with misleading results as they use $W = F\Delta x\cos\theta$ to solve friction problems and for no good reason, since often the desired results could have been properly obtained through the use of Newton’s Second Law.

Describing and representing energy in systems

A careful treatment of energy will keep track of the amounts of energy stored in various entities and will also explicitly identify the energy source and the energy recipient involved in every pertinent energy exchange. Keeping track of the amounts of energy in various entities can be done quite nicely using the energy bar graphs advocated by Alan van Heuvelen.[1] Keeping track of the sources and recipients of energy can be done with energy flow diagrams as prescribed by the faculty of the Abteilung für Didaktik der Physik at the University of Karlsruhe, Germany.[2] Although the Karlsruhe group has developed a thoroughly integrated physics curriculum using energy as the fundamental and unifying concept, we will pursue only a small portion of their agenda.

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[1] Alan van Heuvelen, *ActivPhysics*, Addison-Wesley (Reading, MA, 1995).

[2] G. Falk and F. Herrmann, eds., *Konzepte eines zeitgemäßen Physikunterrichts*, 5 vols., (Schroedel Verlag, Hannover, 1982)

In many introductory courses, energy can be characterized as internal (thermal) energy, kinetic energy, gravitational energy, electrical energy, or elastic energy. In the face of these commonly used names, it is important for students to understand that energy does not come in different forms. The distinctive names arise because of the different *locations* in which energy is stored, not because there are different forms of energy. Energy is just energy. It will also be noticed that the term "potential energy" is not used in this paper in connection with *any* way of storing energy. This is done because it is sometimes said that there is kinetic energy and then there is energy that is stored, namely "potential" energy. Actually all energy is "stored" in some physical entity, not just the so-called "potential" energies. For example, kinetic energy is "stored" in moving particles and in flywheels, etc., and is available for transfer. Therefore, the characterization of some kinds of energy as potential (meaning "stored") and kinetic (not stored?) is artificial and possibly misleading.

It is interesting to point out that gravitational energy cannot be considered to be stored in some elevated object or by the Earth in whose gravitational field the object exists. Students often come to believe that an object itself can possess gravitational energy in the same sense that it can possess kinetic energy or thermal energy. However, whenever the amount of energy in some entity is changed or redistributed, one or more state variables of that entity will change. Since no property at all of an object itself changes as it is raised or lowered, gravitational energy cannot be regarded as being contained in the object. Some argue that gravitational energy is a property of the *system* consisting of the two gravitating bodies under consideration and that their separation constitutes the state variable that changes with the amount of gravitational energy. All well and good. But this claim simply skirts the issue of *where* in the system the energy is actually stored. If the system consists of only the two gravitating bodies, and neither contains the gravitational energy, there would seem to be no place for the energy to exist. The missing part of the system, the gravitational field, needs to be recognized. We will consider gravitational energy to be stored in the gravitational field. More on the gravitational field as a "container" for energy can be found in an article by Stößel.[3]

If we claim that energy is stored in the gravitational field, and if a change in energy content is indicated by some change in the thing that stores it, then what changes in the field when it gains or loses energy? In the Newtonian picture of gravitation, the strength of the gravitational field changes. For example, if a meteor falls to earth there is less gravitational energy than before it fell. But now the earth has a bit more mass and a slightly stronger gravitational field. The stronger the gravitational field, the less gravitational energy it possesses. In general relativity the curvature of space-time depends on the position of objects in space. This curvature, then, is what changes when energy flows to or from a gravitational field. Describing *where* energy is stored in gravitational fields, however, is a problem in general relativity that still occupies physicists.[4,5] This is a nice example of how close the concepts in introductory physics can come to the cutting edge of knowledge.

Why bother with something like the curvature of space-time that students cannot possibly measure? We bother because something is gained, and that is the ability to visualize energy flowing from one thing to another, producing changes in the giver and in the receiver. This enables students to describe energy transfers in terms of meaningful physical entities and the variables that describe them rather than in terms of the meaningless and misleading notion of energy changing from one "form" to another.

Energy bar graphs enable one to describe the state of some system in terms of the energy *stored* in various physical entities within the system. In an energy bar graph one first identifies the system of interest. The system is examined in an

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[3] W. Stößel, "Die Rolle der Energie in der Mechanik," in Falk and Herrmann, eds., *Konzepte eines zeitgemäßen Physikunterrichts*, (Schroedel Verlag, Hannover, 1982), Vol. V, pp. 48 ff.

[4] Misner, Thorne, and Wheeler, *Gravitation*, (Freeman, San Fransisco, 1973), pp. 466 ff., pp. 603 f.

[5] S. Hayward, "Quasi-local gravitational energy," Phys. Rev. D **49** (1994) 831-839.

initial state and in a final state. In the initial state of a system there may be several entities that store (possess) energy. The energy stored by each entity is represented by a vertical bar on a bar graph along with any energy flows that flows into or out of the system through working or heating as the system progresses from the initial state to the final state. In the final state the energy stored by each entity in the system is represented in a new bar graph. In any case, the total amount of energy in the final state of a system must equal the total amount of energy in the initial state plus or minus any energy that was gained or lost through interactions with entities outside the system.

The energy flow within and between systems can be represented by *energy flow diagrams.* Such diagrams will illustrate *how* a system gets from its initial state to its final state. In flow diagrams the energy containers are represented by named rectangles. The flow of energy from one container to another will be represented by an arrow or arrows, the thickness of which corresponds qualitatively to the amount of energy transferred during the transition from the initial state to the final state.

The difficulties associated with the sign convention for work *W* done *by* or performed *on* a system are avoided for beginners by concentrating only on determining whether energy was gained or lost by the system of interest.

Energy bar graphs and flow diagrams involving the free particle model

We will first consider the case of a box being pulled with a rope along a rough surface at a constant speed by a person. Such an object is in the domain of the free particle model. A careless but common application of $W = F\Delta x\cos\theta$ will lead a student to conclude that no work is being done on the box, since the net force on the box is zero. After all, the box has a constant velocity and, therefore, a constant kinetic energy. Or a student may calculate the work done on the box by the person according to $W = F\Delta x\cos\theta$ and then calculate the work done through the force of friction on the box in the same way. Since the frictional force is of the same size as the force exerted by the person but oppositely directed, the student, using the same Δx for the frictional force as for the force exerted by the person, may find that the frictional work just cancels that done by the person. This, of course, leads to inconsistencies that ought to bother a student. The box becomes warmer, indicating that it has gained energy. Therefore the work done on the box is greater than zero. If the student realizes this, then instead of trying to figure out how one can properly use $W = F\Delta x\cos\theta$ for the frictional force (not a trivial task!), a student could use energy bar graphs and flow diagrams to characterize the system qualitatively, more accurately, and more cogently.

Let us use energy bar graphs and an energy flow diagram to represent the person dragging the box in Figure 1. We will define the system to consist of the person, the box, and the floor. The initial state of the system is represented by the energy bar graph on the left. The final state is represented by the graph on the right. And the process by which the system goes from initial to final state is represented by the energy flow diagram in the middle.

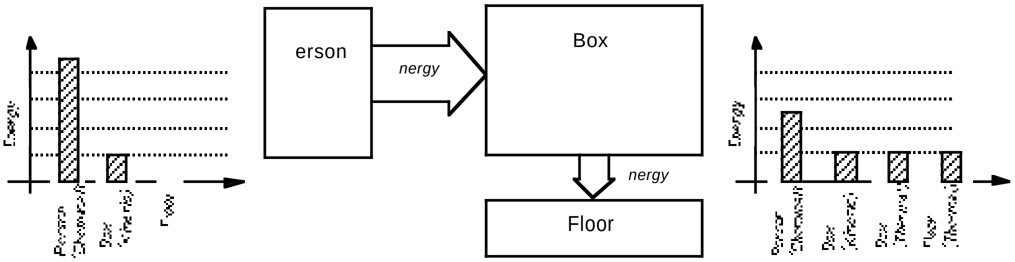

Figure 1: Energy bar charts and energy flow diagram for a box being pushed across a floor

This energy flow diagram is by no means the simplest. The energy put into the box by the person through the rope is transferred at innumerable locations along the rubbing surfaces to the molecular structures of the box and of the floor which we then call “internal” or “thermal” energy. Some of the energy entering the box from the person stays in the box while some goes to the floor. This process is depicted by representing the energy flow from the person to the box as a thicker arrow and the energy flow from the box to the floor as a thinner arrow. The box contains more energy at the end of the process than at the beginning.

It is worth considering the box alone to be the system of interest (Fig. 2). One could indicate this choice on the energy flow diagram by encircling the diagram for the box to represent the system boundary. The graphs and diagrams for the person dragging the box change as shown.

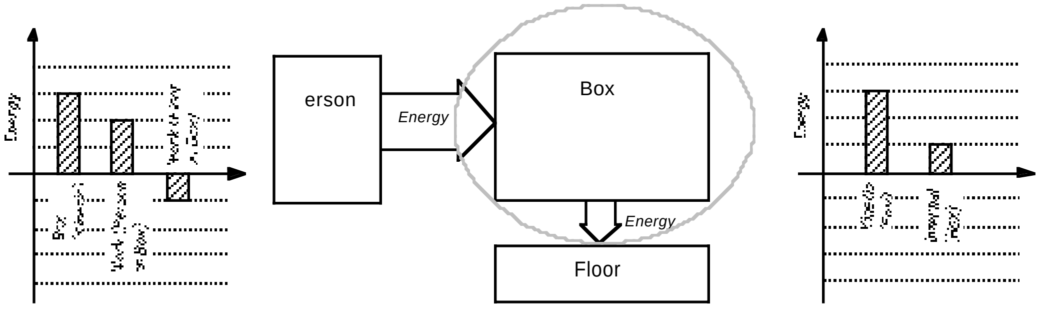

Figure 2: Identifying the box itself as a system

As mentioned above, there is a tendency for students who deal with work exclusively according to $W = F\Delta x\cos\theta$ to claim that there is no net work done on the box. However, net work *is* done on the box. Using the energy flow diagram, we can see that energy flows because of a force from the person across the system boundary to the box. Energy entering a system in this way is called "work." It is not necessary to *call* it work in order to understand what is occurring, however. Also, energy flows across the boundary from the box to the floor because of frictional force, and the floor becomes warmer as a result. This quantity of energy is also "work." And because some internal energy remains in the box as indicated by an increase in temperature, the box has more energy than before. The amount of energy gained by the box is the net amount of work done on it. The error of claiming that there is no net work done on the box is avoided, although there is now no quick and simple way to find out how much work *is* done on the box by the floor through frictional force.

Why does $W = F\Delta x\cos\theta$ fail to tell us the work done by the floor on the box? There must be something subtle about how one uses this relation. These subtleties are discussed in some detail by Sherwood and Bernard.[6] It turns out that the details of how the frictional force arises on the microscopic scale complicate the calculation of work done through friction. At the actual points at which forces are exerted we know neither the sizes of the actual forces nor the distances over which they are acting. Furthermore, $W = F\Delta x\cos\theta$ is defined for a *particle*, and a particle can have no internal energy. $W = F\Delta x\cos\theta$ should not, in general, be used for work done through frictional forces, since a particle model is not a valid model for objects that have changing internal energy and, therefore, internal structure.

Other energy flow diagrams can be simpler to draw and interpret. The next example (Fig. 3) has to do with an object that is being lifted vertically at a constant velocity by a motor. Energy flows from an electrical powerplant across the system boundary because of electrical forces to the motor (so it is called "work") and then to an object, but the energy is not stored in the object. The object is not changed in any way as one would expect were it to store energy in some way. Where, then, is the energy stored? It is stored in the gravitational field itself. We will call this energy "gravitational energy." We avoid calling this energy "the potential energy of the object," because in no way can this energy be considered a property of the object in the same sense that its kinetic energy can be.

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[6] B. A. Sherwood and W. H. Bernard, "Work and heat transfer in the presence of sliding friction," American Journal of Physics, **52**, pp. 1001-1007 (1984).

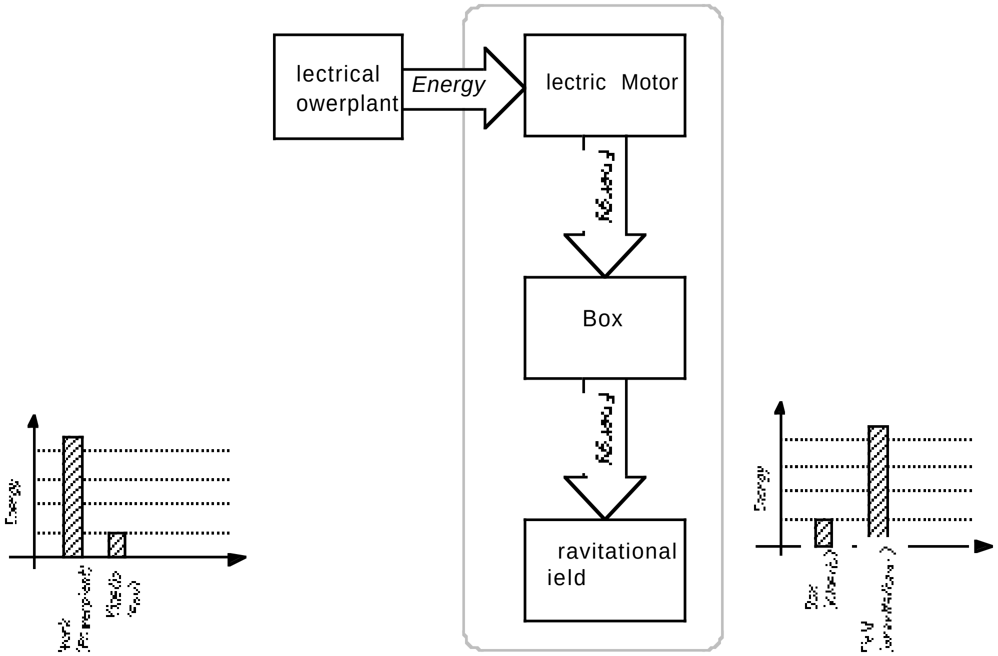

Figure 3: Energy bar charts and energy flow diagrams for a box being lifted by an electric motor

Energy flow diagrams for the constant force particle model

Let us consider an object subject to a single, constant force (but not friction!). In this case energy flowing to or from the object will be stored in the object as kinetic energy. This flowing energy will be exchanged with some other entity as in the case of an object subject to a gravitational force exerted by the earth's gravitational field (Fig. 4). For a freely falling object we can represent the system as follows:

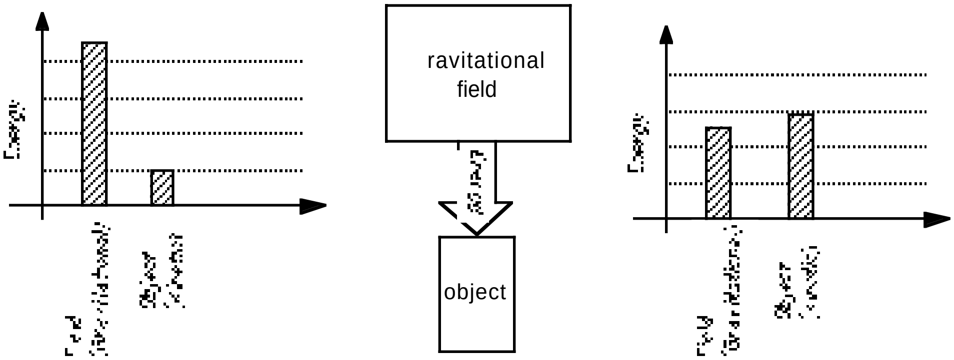

Figure 4: Energy bar charts and energy flow diagram for a freely falling object

In this case energy is transferred from the gravitational field to the object where it is stored as energy of motion, kinetic energy.

In the case of a car stopping with uniform acceleration (Fig. 5) we have the kinetic energy stored in the car being transferred to the car and to the road where it is stored in motions and bonds at the atomic level (thermal energy). One could represent the stopping process by the following bar graphs and flow diagram.

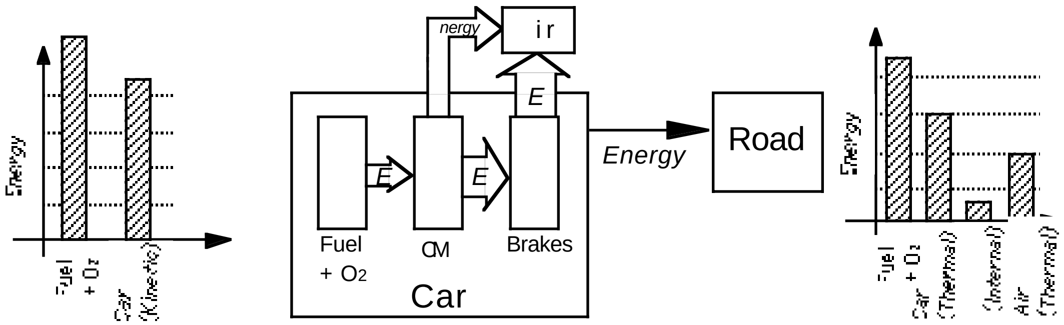

Figure 5: Energy bar charts and energy flow diagram for a car braking to a stop

The initial state of the system is described in the left energy bar graph. The process through which it changes to the final state is represented in the flow diagram. Energy is transferred from the fuel to the engine and to the center of mass (kinetic energy) and then to the brakes (stored in the atomic motions and bonds, *i.e. internal* or *thermal energy*), and also a little flows to the road (thermal energy). Energy then flows from the brakes as they cool and from the hot engine to the air where it is stored in the motions of the molecules. The final state of the system is described in the energy bar graph on the right.

It would be illustrative to consider the car alone as the system of interest. The quantity of energy that leaves the car and flows to the air (crossing the system boundary) through the cooling of the brakes is called “heat,” but it is not necessary to call it heat in order to understand the situation. It is interesting to note that there is very little work done by the car. Because of an inappropriate use of $W = F\Delta x\cos\theta$ or the work-energy theorem a student might think that a lot of work is done *via* the force of the road on the car. In fact, not much energy flows to the road at all!

Energy in electric circuits

That the energy concept is of fundamental importance throughout all of physics goes without saying. The same representations that we have described can be used also in other branches of physics. Let us look at how flow diagrams can be used in a simple electric circuit consisting of a dry cell, copper wire, a lamp, and a resistor connected in series.

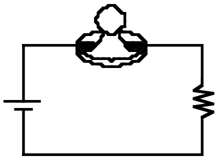

Figure 6: A simple circuit

Usually such circuits are not thought of as being in well defined initial and final states as mechanical systems in introductory physics often are. In general, the flow of current is an ongoing process and so is particularly well suited for representation by an energy flow diagram.

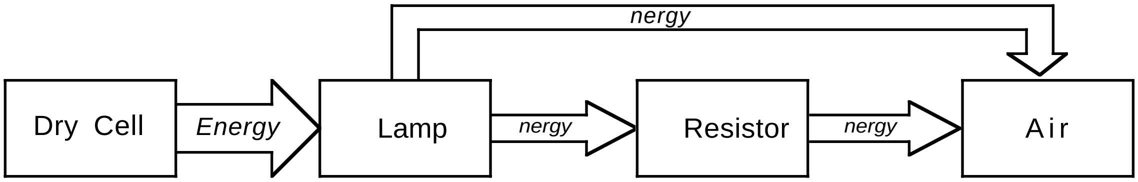

Figure 7: Energy flow diagram for a simple resistive circuit

Initial and final states could certainly be represented by energy bar graphs should one so choose. If one chooses to include only the circuit in the system, then the energy that crosses the system boundary to the air is called heat, though it need not be called heat in order to understand the situation.

Notice that the energy flowing along the circuit diminishes as it encounters resistive elements. Students commonly and mistakenly assert that the charge current itself is consumed as it flows around the circuit. It is actually the energy flowing that is "consumed." This is a case of intuition attaching itself to the wrong concept.

Energy and the particle model of light

The course of energy can be followed for a quantum of light also. Consider the emission and subsequent absorption of some quantum of light. The energy bar graphs will change length only by certain discrete amounts in such exchanges. Photon exchanges more complex than Figure 8 could illustrate this quantum nature of atomic energy clearly.

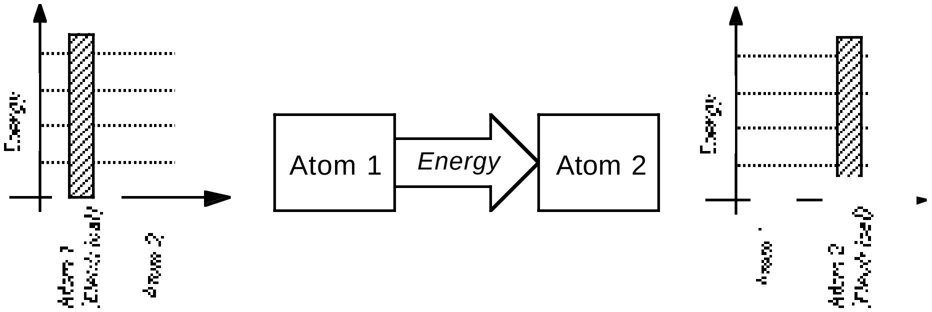

Figure 8: Energy bar charts and energy flow diagram for emission and absorption of a photon

Summary

Student difficulties with the work concept can develop as a result of dealing with work only as $W = F\Delta x\cos\theta$. Since understanding work requires a working understanding of energy and systems, students should first become acquainted with how energy flows in the systems that are of interest in introductory physics. The first law of thermodynamics, $\Delta E = Q - W$ (energy conservation), provides a deeper and simpler insight into the nature of both energy *and* work than $W = F\Delta x\cos\theta$. Energy bar graphs and flow diagrams provide students with richer means for representing energy processes, since they illustrate both energy conservation and work. $W = F\Delta x\cos\theta$ can be applied when it can be used in a straightforward manner and where subtleties involved as with frictional forces do not preclude its use. In this way students can begin to understand not only the appropriate use of this relation but also its practical limitations. The top billing, however, must go explicitly to energy storage and exchanges.